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Conserved quantities in Kerr-anti-de Sitter spacetimes in various dimensions

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ABSTRACT: We compute the conserved charges for Kerr anti-de Sitter spacetimes in various dimensions using the conformal and the counterterm prescriptions. We show that the conserved charge corresponding to the global timelike killing vector computed by the two methods differ by a constant dependent on the rotation parameter and cosmological constant in odd spacetime dimensions, whereas the charge corresponding to the rotational killing vector is the same in either approach. We comment on possible implications of our results to the AdS/CFT correspondence.

KEYWORDS: AdS-CFT Correspondance, Classical Theories of Gravity, Black Holes.

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1. Introduction

Asymptotically anti-de Sitter (AAdS) spacetimes have attracted a great deal of attention recently due to the conjectured AdS/CFT correspondence, which relates supergravity/string theory in bulk AAdS spacetimes to conformal field theories on the boundary, the hope being that a full quantum theory of gravity in AdS spacetimes can be replaced by a well understood CFT/Yang-Mills theory, and observable quantities in the gravity theory can be computed using the latter. A dictionary translating between different quantities in the bulk gravity theory and its counterparts on the boundary has emerged, including the partition functions and correlation functions of both theories.

One of the fundamental set of quantities for any physical theory is the set of conserved quantities associated with it. For theories of gravity on asymptotically flat spacetimes, these are precisely the ADM conserved quantities, which constitute the $d(d+1)/2$ conserved charges corresponding to the Poincaré generators in d-dimensions. However, the ADM formulae break down for spacetimes that are AAdS (i.e. satisfying the Einstein equations with a negative cosmological constant at time-like infinity), implying that a new set of rules have to be laid down to construct conserved quantities corresponding to the asymptotic AdS_d group of isometries. Efforts in this direction were made in [3, 4]. However, the conserved charges constructed had supertranslation-like ambiguities (due to the coordinate dependence of the formalism), or relied on using an auxiliary spacetime, in which the boundary of the AAdS spacetime had to be embedded in a reference spacetime. The latter procedure is neither unique, nor always possible. In [5], the Penrose conformal completion was used to define conserved quantities for $d = 4$, which removed the aforementioned drawbacks. Its generalization to dimensions $d \geq 4$ was done in [6].

Independently, the AdS/CFT correspondence inspired an alternative approach of constructing conserved quantities for AdS spacetimes [7]. This 'counterterm' method proposes certain boundary terms (or counterterms) which depend on the intrinsic geometry of the (timelike) boundary at large spatial distances. These do not affect

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the bulk equations of motion and eliminate the need to embed the given geometry in a reference spacetime. However, this approach still involves taking the the problematic limit $r \to \infty$, where metric components necessarily diverge, unlike the asymptotically flat spacetimes.

Although the above approaches can in principle be used to construct conserved quantities for arbitrary AAdS spacetimes, in practice the conformal formalism has only been used to compute the conserved quantity associated with the global timelike killing vector field (KVF) for Schwarzschild-AdS spacetimes for $d = 4$ and 5 (we will call this quantity the mass, in analogy with the its counter part for asymptotically flat geometries, although the group is no longer Poincaré). On the other hand, the counterterm method was used to calculate the conserved quantities for Schwarzschild as well as Kerr-AdS (KAdS) geometries in four dimensions [8] as well as for $d \leq 7$ [9]

In this paper we carry out a systematic analysis of KAdS black holes in arbitrary dimensions $d \geq 4$ using the conformal as well as the counterterm formalisms. We employ the algorithm of Kraus, Larsen and Siebelink (KLS) [10] to construct the boundary counterterm contributions up to $d = 9$, and from this derive the action, mass and angular momentum for $4 \leq d \leq 9$. We will show that for $d =$ odd, the mass of the spacetime depends on which formalism we use and their difference is the so called 'Casimir energy', which is a function of Λ, the cosmological constant and a, the angular momentum parameter. On the other hand, the angular momentum of these solutions is independent of the method of computation, and hence unambiguous. In either case, our results are commensurate with the Gibbs-Duhem relation in semiclassical euclidean quantum gravity, which relates one-quarter of the area of the event horizon to the minus the difference between the (euclidean) action and the hamiltonian times the inverse temperature.

2. The conformal and counterterm methods

Let us begin with a brief review of the methods under consideration. In the conformal method, one begins with the following assumptions regarding the physical d dimensional AAdS manifold $\hat{M}$ with metric $\hat{g}_{ab}$ and the conformally mapped manifold M with metric g_{ab}:

(i) $g_{ab} = \Omega^2 \hat{g}_{ab}$, where Ω is a non-negative conformal factor.

(ii) the boundary $\mathcal{I}$ of M is topologically $S^{(d-2)} \times R$, Ω vanishes on $\mathcal{I}$, but $\nabla_a \Omega \neq 0$ on $\mathcal{I}$.

(iii) The Einstein equations with $\Lambda < 0$ are satisfied on $\mathcal{I}$.

(iv) The fall-off behaviors of the matter fields and the Weyl tensor are such that $\Omega^{2-d} T_{ab}$ and $\Omega^{4-d} C_{abcd}$ are smooth on $\mathcal{I}$.

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It can be easily verified that the d-dimensional KAdS spacetime satisfies the above conditions. Then transforming all tensors to the conformally mapped space-time, and after a series of straightforward analyses, it can be shown that the following equation is valid on $\mathcal{I}$:

$$D^p \mathcal{E}_{mp} = -8\pi \frac{(d-3)}{\ell} \mathbf{T}_{ab} n^a h^b{}_m \,, \tag{2.1}$$

where D^d is the intrinsic covariant derivative on $\mathcal{I}$, compatible with the induced metric $h_{ab} := g_{ab} - \ell^2 n_a n_b$ ($n_a := \nabla_a \Omega$) and $\mathcal{E}_{ab}$ is the electric part of the Weyl tensor at $\mathcal{I}$ defined as: $\mathcal{E}_{ab} := \ell^2 \Omega^{3-d} C_{ambn} n^m n^n$ (we have set Newton's constant to be unity here and in the following analyses). We have parametrized the cosmological constant as $\Lambda = -(d-1)(d-2)/2\ell^2$ and $\mathbf{T}_{ab} := \Omega^{2-d} \hat{T}_{ab}$ on $\mathcal{I}$. From the above equation, the conserved charge associated with the conformal KVF ξ is defined as follows (note that an ordinary KVF on $\hat{M}$ becomes the conformal KVF on M):

$$Q_\xi[C] := -\frac{1}{8\pi} \frac{\ell}{d-3} \oint_C \mathcal{E}_{ab} \xi^a dS^b \,, \tag{2.2}$$

which satisfies the balance equation

$$Q_\xi[C_2] - Q_\xi[C_1] = \int_{\Delta \mathcal{I}} \mathbf{T}_{ab} \xi^a dS^b \tag{2.3}$$

in presence of matter fields (C_1 and C_2 are two cross sections on $\mathcal{I}$). Equations (2.2) and (2.3) are the fundamental relations which we will use to define conserve quantities.

Now we write down the explicit KAdS family of solutions. For simplicity we will assume only one rotation parameter [11]:

$$\begin{aligned} ds^2 = & -\frac{\Delta_r}{\rho^2} \left(dt - \frac{a}{\Xi} \sin^2\theta d\phi \right)^2 + \frac{\rho^2}{\Delta_r} dr^2 + \frac{\rho^2}{\Delta_\theta} d\theta^2 + \\ & + \frac{\Delta_\theta \sin^2\theta}{\rho^2} \left[a dt - \frac{(r^2+a^2)}{\Xi} d\phi \right]^2 + r^2 \cos^2\theta d\Omega^2_{d-4} \,, \end{aligned} \tag{2.4}$$

where

$$\begin{aligned} \Delta_r &= \left(r^2 + a^2 \right) \left(1 + \frac{r^2}{l^2} \right) - 2mr^{5-d} \,, \\ \Delta_\theta &= 1 - a^2 \cos^2 \frac{\theta}{l^2} \,, \\ \Xi &= 1 - \frac{a^2}{l^2} \,, \\ \rho^2 &= r^2 + a^2 \cos^2\theta \,, \end{aligned} \tag{2.5}$$

and $d\Omega^2_{d-4}$ is the line element on unit S^{d-4}. The parameters m and a are related to the mass and angular momentum of the black hole, as we shall see.

Evaluation of the electric part of the Weyl tensor (at $\mathcal{I}$) yields to leading order in r. See table 1.

Dim	$\mathcal{E}_{tt}$	$\mathcal{E}_{t\phi}$
4	$-2m/l^2r$	$2\sin^2\theta ma/\Xi l^2 r$
5	$-6m/l^2r^2$	$6\sin^2\theta ma/\Xi l^2 r^2$
6	$-12m/l^2r^3$	$12\sin^2\theta ma/\Xi l^2 r^3$
7	$-20m/l^2r^4$	$20\sin^2\theta a/\Xi l^2 r^4$
8	$-30m/l^2r^5$	$30\sin^2\theta ma/\Xi l^2 r^5$
9	$-42m/l^2r^6$	$42\sin^2\theta ma/\Xi l^2 r^6$

Table 1: Components of the Electric Weyl tensor to leading order in 1/r.

To evaluate the surface element dS^b, we use the determinant of the (hypothetical) induced metric on C:

$$\sigma_{ab} := g_{ab} - l^2 n_a n_b + u_a u_b\,, \tag{2.6}$$

where u^a is the timelike unit normal at C.

Finally, using the timelike KVF $\partial/\partial t$ and the rotational KVF $\partial/\partial\phi$, and transforming back the integrals at $\mathcal{I}$ to the physical space time $\hat{M}$, the mass and angular momenta of the KAdS spacetime in various dimensions can be calculated. The results are presented in table 2.

Now we move on to the counterterm action and the conserved charges obtained thereof. As is well known, the Einstein action with a negative cosmological constant along with the Gibbons-Hawking boundary term (in the remaining analysis we will always work in the physical spacetime, and omit the hats over the geometrical quantities, for brevity)

$$I = \frac{1}{8\pi G_{(d)}}\left(\int_M d^dx\sqrt{-g}\,[R-2\Lambda] - \int_{\partial M} d^{d-1}x\sqrt{-\gamma}K(\gamma)\right) \tag{2.7}$$

(where K is the trace of the extrinsic curvature of the timelike boundary and γ_{ab} is the induced metric on this boundary) is divergent for the spacetimes under consideration, with fall off conditions natural to this setting. To eliminate this divergence, the KLS counterterm proposal prescribes adding terms to the action which are *intrinsic* to the boundary as $r\to\infty$. These take the form [7]–[10]

$$I_{\mathrm{ct}} = \frac{1}{8\pi G_{(d)}}\int_{\partial M}\mathcal{L}_{\mathrm{ct}}\,, \tag{2.8}$$

where the quantity

$$\tilde{\Pi}_{ab} \equiv \frac{2}{\sqrt{-\gamma}}\frac{\delta S_{\mathrm{ct}}}{\delta\gamma^{ab}} \tag{2.9}$$

is a solution to the Gauss-Coddacci relations

$$\frac{1}{d-2}\tilde{\Pi}^2 - \tilde{\Pi}_{ab}\tilde{\Pi}^{ab} = \frac{(d-1)(d-2)}{\ell^2} + R \tag{2.10}$$

as a power series in $1/\ell$, i.e. $\tilde{\Pi}_{ab} = \frac{1}{\ell}\sum_{n=0}\ell^{2n}\,\tilde{\Pi}^{(n)}_{ab}$. To order n, the relation (2.10) is linear in the trace $\tilde{\Pi}^{(n)}$, and so this quantity may be determined in terms of the lower-order terms. Under local Weyl rescalings, the relation (2.9) ensures that

$$\mathcal{L}^{(n)}_{\mathrm{ct}} = \frac{\tilde{\Pi}^{(n)}}{d-1-2n} \tag{2.11}$$

and by then by using (2.9) again, the full expression for $\tilde{\Pi}_{ab}$ can be obtained to the desired order.

This procedure yields

$$\begin{aligned}
\mathcal{L}_{\rm ct} =& -\frac{d-2}{\ell}\sqrt{-\gamma} - \frac{\ell\sqrt{-\gamma}}{2(d-3)}R - \frac{\ell^3\sqrt{-\gamma}}{2(d-3)^2(d-5)}\left(R^{ab}R_{ab} - \frac{d-1}{4(d-2)}R^2\right) + \\
&+\frac{\ell^5\sqrt{-\gamma}}{(d-3)^3(d-5)(d-7)}\left(\frac{3d-1}{4(d-2)}RR^{ab}R_{ab} - \frac{(d-1)\,(d+1)}{16(d-2)^2}R^3 - \right.\\
&\qquad - 2R^{ab}R^{cd}R_{acbd} + \frac{d-3}{2(d-2)}R^{ab}\nabla_a\nabla_b R - \\
&\qquad \left. - R^{ab}\nabla^2 R_{ab} + \frac{1}{2(d-2)}R\nabla^2 R\right)
\end{aligned} \tag{2.12}$$

to order ℓ^5. All the geometrical quantities above are intrinsic to the timelike boundary at $r\to\infty$). An integration by parts renders this expression in the more convenient form

$$\begin{aligned}
\mathcal{L}_{\rm ct} =& -\frac{d-2}{\ell}\sqrt{-\gamma} - \frac{\ell\sqrt{-\gamma}}{2(d-3)}R - \frac{\ell^3\sqrt{-\gamma}}{2(d-3)^2(d-5)}\left(R^{ab}R_{ab} - \frac{d-1}{4(d-2)}R^2\right) + \\
&+\frac{\ell^5\sqrt{-\gamma}}{(d-3)^3(d-5)(d-7)}\left(\frac{3d-1}{4(d-2)}RR^{ab}R_{ab} - \frac{(d-1)\,(d+1)}{16(d-2)^2}R^3 - \right.\\
&\qquad \left. -2R^{ab}R^{cd}R_{acbd} - \frac{d-1}{4(d-2)}\nabla_a R\nabla^a R + \nabla^c R^{ab}\nabla_c R_{ab}\right).
\end{aligned} \tag{2.13}$$

Varying the action with respect to the boundary metric γ_{ab}, the full stress-energy tensor for gravity is defined as:

$$T_{ab} := \frac{2}{\sqrt{-\gamma}}\frac{\delta}{\delta\gamma^{ab}}\left(S + S_{\rm ct}\right), \tag{2.14}$$

which results in the boundary stress-energy:

$$\begin{aligned}
T_{ab} =& K_{ab} - \gamma_{ab}K - \frac{d-2}{l}\gamma_{ab} + \frac{l}{d-3}\left(R_{ab} - \frac{1}{2}\gamma_{ab}R\right) + \\
&+\frac{l^3}{(d-3)^2(d-5)}\left\{-\frac{1}{2}\gamma_{ab}\left(R_{cd}R^{cd} - \frac{(d-1)}{4(d-2)}R^2\right) - \frac{(d-1)}{2(d-2)}RR_{ab} + \right.\\
&\left. + 2R^{cd}R_{cadb} - \frac{d-3}{2(d-2)}\nabla_a\nabla_b R + \nabla^2 R_{ab} - \frac{1}{2(d-2)}\gamma_{ab}\nabla^2 R\right\} - \\
&-\frac{2\ell^5\sqrt{-\gamma}}{(d-3)^3(d-5)(d-7)}\left\{\frac{3d-1}{4(d-2)}\left[\left(G_{ab}R^{cd}R_{cd}\right) - \nabla_a\nabla_b\left(R^{ef}R_{ef}\right) + \right.\right.\\
&\quad + \gamma_{ab}\nabla^2\left(R^{ef}R_{ef}\right) + 2RR_a^{\ c}R_{bc} + \gamma_{ab}\nabla_c\nabla_d\left(RR^{cd}\right) + \\
&\quad \left. + \nabla^2\left(RR_{ab}\right) - \nabla^c\nabla_b\left(RR_{ac}\right) - \nabla^c\nabla_a\left(RR_{bc}\right)\right] - \\
&- \frac{(d-1)\,(d+1)}{16(d-2)^2}\left[-\frac{1}{2}\gamma_{ab}R^3 + 3R^2R_{ab} - 3\nabla_a\nabla_b R^2 + 3\gamma_{ab}\nabla^2 R^2\right] -
\end{aligned}$$

$$
\begin{aligned}
&-2\Big[-\frac{1}{2}\gamma_{ab}R^{ef}R^{cd}R_{ecfd}+\frac{3}{2}\left(R_a^{\ e}R^{cd}R_{ecbd}+R_b^{\ e}R^{cd}R_{ecad}\right)-\nabla_c\nabla_d\left(R_{ab}R^{cd}\right)+\\
&\quad+\nabla_c\nabla_d\left(R_a^{\ c}R_b^{\ d}\right)+\gamma_{ab}\nabla_e\nabla^f\left(R^{cd}R^e_{\ cfd}\right)+\nabla^2\left(R^{cd}R_{acbd}\right)-\\
&\quad-\nabla_e\nabla_a\left(R^{cd}R^e_{\ cbd}\right)-\nabla_e\nabla_b\left(R^{cd}R^e_{\ cad}\right)\Big]-\\
&-\frac{d-1}{4(d-2)}\Big[\nabla_a R\nabla_b R-\frac{1}{2}\gamma_{ab}\Big(\nabla_c R\nabla^c R\Big)-2R_{ab}\nabla^2 R\\
&\qquad\qquad-2\gamma_{ab}\nabla^4 R+2\nabla_a\nabla_b\nabla^2 R\Big]+\\
&+\Big[2\nabla_c R_{ad}\nabla^c R_b^{\ d}+\nabla_a R^{cd}\nabla_b R_{cd}-\frac{1}{2}\gamma_{ab}\nabla^e R^{cd}\nabla_e R_{cd}-\gamma_{ab}\nabla_c\nabla_d\nabla^2 R^{cd}-\\
&\quad-\nabla^4 R_{ab}+\nabla_c\nabla_a\nabla^2 R_b^{\ c}+\nabla_c\nabla_b\nabla^2 R_a^{\ c}-\nabla_c\left(R_{bd}\nabla_a R^{cd}\right)-\\
&\quad-\nabla_c\left(R_{ad}\nabla_b R^{cd}\right)-\nabla_c\left(R_{ad}\nabla^c R_b^{\ d}+R_{bd}\nabla^c R_a^{\ d}\right)+\\
&\quad+\nabla^c\left(R_{cd}\nabla_a R_b^{\ d}\right)+\nabla^c\left(R_{cd}\nabla_b R_a^{\ d}\right)\Big]\Big\}\,,
\end{aligned}
\tag{2.15}
$$

which is valid for all $4 \leq d \leq 9$. For a given dimension d, all terms of order greater than $\ell^{[d/2+1]}$ give vanishing contributions to the conserved charges at infinity, where $[n/2]$ is the largest integer less than $n/2$. We do a case by case analysis to determine the relevant terms in any specific spacetime dimension.

Here, apart from the intrinsic curvature terms, one evaluates the extrinsic curvature K_{ab} of the boundary. The final expression for the conserved charge associated with an (ordinary) KVF ξ is, in this case:

$$
Q_\xi := \int_\sigma d^{d-2}x\sqrt{\sigma}T_{ab}u^a\xi^b\,, \tag{2.16}
$$

where u^a is the timelike unit normal at the boundary cylinder at $r \to \infty$ and $\sigma_{ab} := \gamma_{ab} - u_a u_b$.

We have computed from (2.15) the components of T_{ab} for $d = 4, 5, 6, 7, 8, 9$. Our results for $d \leq 7$ are in agreement with previous results [8, 9, 12]; since the expression are somewhat long and cumbersome (especially for the last two cases) we shall not reproduce them here. Inserting these expressions into (2.16) for the curvature tensors and the extrinsic curvatures, we finally obtain the values of the timelike and rotational conserved charges for KAdS spacetimes in various dimensions. The results also appear in table 2, for easy comparison with the conformal case. The masses computed from the conformal method and that from the CFT method are given in units of π/Ξ and the angular momentum in units of $am\pi/\Xi^2$. The angular momenta as calculated with either method yield the same values, and so we have provided only one column for this case. The masses differ for d =odd because of the presence of the Casimir energy induced by the extra boundary terms.

Dim	Mass (conformal)	Mass (counterterm)	Angular Momentum
4	$\frac{m}{\pi}$	$\frac{m}{\pi}$	$\frac{1}{\pi}$
5	$\frac{3m}{4}$	$\frac{3}{4}m + \frac{9\ell^4 - 9a^2\ell^2 + a^4}{96\ell^2}$	$\frac{1}{2}$
6	$\frac{4m}{3}$	$\frac{4m}{3}$	$\frac{2}{3}$
7	$\frac{5m\pi}{8}$	$\frac{5\pi m}{8} - \frac{\pi(a^6 + 5a^4\ell^2 - 50a^2\ell^4 + 50\ell^6)}{1280\ell^2}$	$\frac{\pi}{4}$
8	$\frac{4\pi m}{5}$	$\frac{4\pi m}{5}$	$\frac{4\pi}{15}$
9	$\frac{7m\pi^2}{24}$	$\frac{7m\pi^2}{24} - \frac{\pi^2(3a^8 - 41a^6\ell^2 - 87a^4\ell^4 + 1225a^2\ell^6 - 1225\ell^8)}{107520\ell^2}$	$\frac{\pi^2}{12}$

Table 2: Evaluation of mass and angular momentum at infinity for the KAdS spacetimes using both counterterm and conformal methods.

We have also cross-checked these results with the Gibbs-Duhem relation [13], which states that

$$S = \beta H_\infty - I\,, \tag{2.17}$$

where I is the euclidean space action and $H_\infty = M - \Omega J$, where M and J are respectively the mass and angular momentum as computed by either method and $\Omega = \frac{a\Xi}{r_+^2+a^2}$ is the angular velocity at the horizon for all dimensionalities. The entropy S is given by one-quarter of the horizon area for the KAdS spacetimes. In table 3 we list the results of a computation of the action using equations (2.8) and (2.12), along with the values for the inverse temperature and entropy. The boxed terms for each of the odd-dimensional cases are the extra terms in the action induced by the counterterm contributions; these terms would are absent if we compute the action using the methods of refs. [3, 4].

3. Discussion

As can be observed from table 3, the expressions for both the mass and the action obtained from the conformal and the counterterm prescriptions do not agree in odd dimensionalities, whereas the angular momenta from the two approaches agree for all dimensions. However, the disagreements are precisely such that the extra (or Casimir) energies exactly balance the additional action contributions and the Gibbs-Duhem relation is still satisfied.

Several comments are in order here. The difference in mass for for $d = 5$ calculated from the two different methods was interpreted as the Casimir energy of $\mathcal{N} = 4\,\mathrm{SU}(N)$ Yang-Mills theory on the conformal boundary $S^3 \times R$ of AdS_5. It is not evident that the differences for $d = 7$ and 9 lend themselves to an analogous

d	Inverse Temperature	Entropy	Action
4	$\dfrac{4\pi\ell^2 r_+\left(r_+^2+a^2\right)}{\left(r_+^2\ell^2+3r_+^4+a^2r_+^2-a^2\ell^2\right)}$	$\dfrac{\pi\left(r_+^2+a^2\right)}{\Xi}$	$-\dfrac{\pi\left(r_+^2+a^2\right)^2\left(r_+^2-\ell^2\right)}{\Xi\left(r_+^2\ell^2+3r_+^4+a^2r_+^2-a^2\ell^2\right)}$
5	$\dfrac{2\pi\ell^2\left(r_+^2+a^2\right)}{r_+\left(\ell^2+2r_+^2+a^2\right)}$	$\dfrac{\pi^2 r_+\left(r_+^2+a^2\right)}{2\Xi}$	$-\dfrac{\pi^2\left(r_+^2+a^2\right)}{48\Xi r_+\left(\ell^2+2r_+^2+a^2\right)}$ $\times\left(12\left(r_+^2+a^2\right)\left(r_+^2-\ell^2\right)\right.$ $\left.+\left[9\ell^2\left(a^2-\ell^2\right)-a^4\right]\right)$
6	$\dfrac{4\pi\ell^2 r_+\left(r_+^2+a^2\right)}{\left(3r_+^2\ell^2+5r_+^4+3a^2r_+^2+a^2\ell^2\right)}$	$\dfrac{2\pi^2r_+^2\left(r_+^2+a^2\right)}{3\Xi}$	$-\dfrac{2\pi^2r_+^2\left(r_+^2+a^2\right)^2\left(r_+^2-\ell^2\right)}{3\Xi\left(3r_+^2\ell^2+5r_+^4+3r_+^2a^2+a^2\ell^2\right)}$
7	$\dfrac{2\pi\ell^2 r_+\left(r_+^2+a^2\right)}{\left(2r_+^2\ell^2+3r_+^4+2a^2r_+^2+a^2\ell^2\right)}$	$\dfrac{\pi^3r_+^3\left(r_+^2+a^2\right)}{4\Xi}$	$-\dfrac{\pi^3r_+\left(r_+^2+a^2\right)}{640\Xi\left(2r_+^2\ell^2+3r_+^4+2r_+^2a^2+a^2\ell^2\right)}$ $\times\left(80r_+^2\left(r_+^2+a^2\right)\left(r_+^2-\ell^2\right)\right.$ $\left.-\left[50\ell^4\left(a^2-\ell^2\right)-a^4\left(a^2+5\ell^2\right)\right]\right)$
8	$\dfrac{4\pi\ell^2 r_+\left(r_+^2+a^2\right)}{\left(5r_+^2\ell^2+7r_+^4+5a^2r_+^2+3a^2\ell^2\right)}$	$\dfrac{4\pi^3r_+^4\left(r_+^2+a^2\right)}{15\Xi}$	$-\dfrac{4\pi^3r_+^4\left(r_+^2+a^2\right)^2\left(r_+^2-\ell^2\right)}{15\Xi\left(5r_+^2\ell^2+7r_+^4+5a^2r_+^2+3a^2\ell^2\right)}$
9	$\dfrac{2\pi\ell^2 r_+\left(r_+^2+a^2\right)}{\left(3r_+^2\ell^2+4r_+^4+3a^2r_+^2+2a^2\ell^2\right)}$	$\dfrac{\pi^4r_+^5\left(r_+^2+a^2\right)}{12\Xi}$	$-\dfrac{4\pi^4\left(r_+^2+a^2\right)r_+}{53760\Xi\left(3r_+^2\ell^2+4r_+^4+3r_+^2a^2+2a^2\ell^2\right)}$ $\times\left(2240r_+^4\left(r_+^2+a^2\right)\left(r_+^2-\ell^2\right)\right.$ $-\left[1225\ell^6\left(a^2-\ell^2\right)\right.$ $\left.\left.-a^4\left(3a^4-41a^6\ell^2-87\ell^4\right)\right]\right)$

Table 3: Evaluation of the entropy and action for the KAdS spacetimes using the counterterm method.

interpretation, because in these cases, the boundary CFTs are not well understood. The additional contribution to the mass is a decreasing positive function of a/ℓ over the allowed range $|a| \leq \ell$ for $d = 5, 9$ whereas it is an increasing negative function of a/ℓ over this range for $d = 7$, so the CFT in the latter case must have a negative Casimir energy. Both methods are consistent with the Gibbs-Duhem relation (2.17); in the counterterm method, the additional contributions to the action from the counterterms are exactly canceled by the Casimir contributions to the mass. None of the additional contributions from the counterterms have a well-defined flat space limit; both the mass and the action diverge as $\ell \to \infty$.

It is interesting to note that there is no corresponding 'Casimir' contribution for the rotational KVF. Perhaps a clearer understanding of this is required in the light of the AdS/CFT correspondence. An interesting check would be to compute the other conserved charges for multiple rotational parameters for $d \geq 5$ and see whether there are Casimir like terms for these charges. Note that from the purely general relativistic point of view, any such terms can be ruled out by simple yet robust covariance arguments [6]. It may be noted that the counterterms become more and more complicated as the dimensionality of the spacetime increases, although they can be uniquely fixed by requiring the elimination of divergences. On the other hand, the conformal method fixes the expressions for the conserved charges once and for all for all d and their explicit computations boil down to the computation of the

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Weyl curvature and the surface element on $\mathcal{I}$. An interesting exercise would be to calculate the conserved charges using both the methods for more complicated AAdS spacetimes like Taub-NUT-AdS and Taub-Bolt-AdS metrics. We hope to report on it elsewhere.

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